Chapter 1

Soustavy linearnich diferencialnich rovnic s konstantnimi koeficienty

(α) Napiseme zadani ulohy a nasledne sestavime prislusnou rozsirenou λ-matici dane ulohy. Pri sestavovani λ-matice i v dalsich krocich pristupujeme k prenasobeni lambdou jako k derivaci. Tedy napriklad

$$(\lambda^2 - 2\lambda + 3)g(t) = g''(t) - 2g'(t) + 3g(t).$$

V pripade, ze resime soustavu

$$\boldsymbol{x}'(t) = \mathbb{A}\boldsymbol{x}(t) + \boldsymbol{b}(t), \tag{1.1}$$

pak nasi λ-matici je matice $(\lambda\mathbb{I} - \mathbb{A}|\boldsymbol{b}(t))$, kde $\mathbb{A} \in M(n \times n)$, $\boldsymbol{b}(t) : \mathbb{R} \to \mathbb{R}^n$.

(β) Pomoci radkovych uprav λ-matice (je zakazano prenasobovat radek nekonstantnim polynomem $P(\lambda)$) upravime matici z predchoziho kroku na rozsirenou horni trojuhelnikovou (nebo matici, ze ktere se da pomoci prehazeni radku a sloupcu vytvorit horni trojuhelnikova viz priklad 8) λ-matici $(\mathbb{B}(\lambda)|\boldsymbol{c})$ (na prave strane rovnice je prenasobeni lambdou brano jako derivace). Na diagonale by nakonci mely byt nenulove polynomy. V pripade, ze resime soustavu (1.1), tak soucet stupnu polynomu na diagonale by mel byt roven n (viz V31).

(γ) Reseni nasi soustavy oznacime $\boldsymbol{x} = (x_1, \dots, x_n)$. Pak postupne hledame funkce $x_n, x_{n-1}, \dots, x_1$ nasledujicim zpusobem. Pokud jsme nalezli $x_n, \dots, x_{m+1}$ pro nejake $m \in \{1, \dots, n\}$ (tedy napr. pro m=n jsme nenalezli nic), pak vypocteme x_m za pouziti m-teho radku nasi upravene matice $(\mathbb{B}(\lambda)|\boldsymbol{c})$ nasledovne. Necht $\mathbb{B}(\lambda) = (b_{i,j}(\lambda))_{i,j=1}^n$. Pocitame linearni diferencialni rovnici s charakteristickym polynomem $\chi(\lambda) = b_{m,m}(\lambda)$ a pravou stranou

$$f(t) = c(t) - \sum_{k=m+1}^{n} b_{m,k}(\lambda)x_k(t),$$

kde prenasobeni λ se pocita jako derivace.

(δ) Zapsani obecneho vysledku a pripadne dopocitani pocatecni podminky.

1.1

(α)

Resime soustavu

$$\begin{aligned} z' + y &= \quad 0, \\ z' - y' &= \quad 3z + y. \end{aligned}$$

Odpovidajici λ-matice tedy je $\left(\begin{array}{cc|c} 1 & \lambda & 0 \\ -1-\lambda & \lambda - 3 & 0 \end{array}\right)$.

(β)

$$\begin{array}{r} I: \\ II + (1+\lambda)I: \end{array} \left(\begin{array}{cc|c} 1 & \lambda & 0 \\ 0 & \lambda^2 + 2\lambda - 3 & 0 \end{array}\right).$$

(γ)

(1) Vypocet $z(t)$: Z 2. radku vidime, ze z splnuje nasledujici rovnici

$$z'' + 2z' - 3z = 0.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice vyssiho radu.

(i) $\chi(t) = t^2 + 2t - 3$,

(ii) $\{-3, 1\}$,

(iii) $\left\{e^{-3t}, e^t\right\}$,

(iv) $z(t) = Ae^{-3t} + Be^t$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$.

(2) Vypocet $y(t)$: Z 1. radku vidime, ze

$$y(t) = -z' = 3Ae^{-3t} - Be^t.$$

(δ)

$\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 3Ae^{-3t} - Be^t \\ Ae^{-3t} + Be^t \end{pmatrix}$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$.

1.2

(α)

Resime soustavu

$$\begin{aligned} 5z' - 2y' + 4z - y &= \quad e^{-t}, \\ z' + 8z - 3y &= \quad 5e^{-t}. \end{aligned}$$

Odpovidajici λ-matice tedy je $\left(\begin{array}{cc|c} -2\lambda - 1 & 5\lambda + 4 & e^{-t} \\ -3 & \lambda + 8 & 5e^{-t} \end{array}\right)$.

(β)

$$\begin{array}{r} II: \\ -3I + (2\lambda+1)II: \end{array} \left(\begin{array}{cc|c} -3 & \lambda + 8 & 5e^{-t} \\ 0 & 2\lambda^2 + 2\lambda - 4 & -8e^{-t} \end{array}\right).$$

(γ)

(1) Vypocet $z(t)$: Z 2. radku vidime, ze z splnuje nasledujici rovnici

$$z'' + z' - 2z = -4e^{-t}.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice vyssiho radu.

(i) $\chi(t) = t^2 + t - 2$,

(ii) $\{-2, 1\}$,

(iii) $\{e^{-2t}, e^t\}$,

(iv) $z_h(t) = Ae^{-2t} + Be^t$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$.

(v) $m = 0, \mu = -1, \nu = 0, k = 0$.

(vi) $R(t) = a$, $z_p(t) = ae^{-t}$, kde $a \in \mathbb{R}$, $t \in \mathbb{R}$.

(vii) $a = 2$.

(xiii) $z(t) = 2e^{-t} + Ae^{-2t} + Be^t$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$.

(2) Vypocet $y(t)$: Z 1. radku vidime, ze

$$y(t) = \frac{1}{3}\left(z' + 8z - 5e^{-t}\right) = 3e^{-t} + 2Ae^{-2t} + 3Be^t.$$

(δ)

$$\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} 3e^{-t} + 2Ae^{-2t} + 3Be^t \\ 2e^{-t} + Ae^{-2t} + Be^t \end{pmatrix},\ A, B \in \mathbb{R},\ t \in \mathbb{R}.$$

1.3

(α)

Resime soustavu

$$\begin{aligned} z' + 3z + y &= 0, \\ y' - z + y &= 0. \end{aligned}$$

Odpovidajici λ-matice tedy je $\left(\begin{array}{cc|c} 1 & \lambda + 3 & 0 \\ \lambda + 1 & -1 & 0 \end{array}\right)$. Navic chceme, aby reseni splnovalo pocatecni podminku $\begin{pmatrix} y(0) \\ z(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(β)

$$\begin{array}{r} I: \\ II - (\lambda+1)I: \end{array} \left(\begin{array}{cc|c} 1 & \lambda + 3 & 0 \\ 0 & -\lambda^2 - 4\lambda - 4 & 0 \end{array}\right).$$

(γ)

(1) Vypocet $z(t)$: Z 2. radku vidime, ze z splnuje nasledujici rovnici

$$z'' + 4z' + 4z = 0.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice vyssiho radu.

(i) $\chi(t) = t^2 + 4t + 4$,
(ii) $\{-2, -2\}$,
(iii) $\{e^{-2t}, te^{-2t}\}$,
(iv) $z(t) = e^{-2t}(A + Bt),\ A, B \in \mathbb{R},\ t \in \mathbb{R}$.

(2) Vypocet $y(t)$: Z 1. radku vidime, ze

$$y(t) = -z' - 3z = e^{-2t}(-A - B - Bt).$$

(δ)

Obecne reseni je $\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = e^{-2t} \begin{pmatrix} -A - B - Bt \\ A + Bt \end{pmatrix}$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$. Pocatecni podminka: $\begin{pmatrix} y(0) \\ z(0) \end{pmatrix} = \begin{pmatrix} -A - B \\ A \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Tedy $A = 1$ a $B = -2$.

1.4

(α)

Resime soustavu

$$\begin{aligned} z'' + y' + z &= e^t, \\ z' + y'' &= 1. \end{aligned}$$

Odpovidajici λ-matice tedy je $\left(\begin{array}{cc|c} \lambda & \lambda^2 + 1 & e^t \\ \lambda^2 & \lambda & 1 \end{array}\right)$.

(β)

$$\begin{array}{r} I: \\ II - \lambda I: \end{array} \left(\begin{array}{cc|c} \lambda & \lambda^2 + 1 & e^t \\ 0 & -\lambda^3 & 1 - e^t \end{array}\right).$$

(γ)

(1) Vypocet $z(t)$: Z 2. radku vidime, ze z splnuje nasledujici rovnici

$$z''' = e^t - 1.$$

Preintegrovanim dostaneme

$$z(t) = e^t - \frac{1}{6}t^3 + At^2 + Bt + C,\ \ A, B, C \in \mathbb{R},\ \ t \in \mathbb{R}.$$

(2) Vypocet $y(t)$: Z 1. radku vidime, ze

$$y'(t) = e^t - z'' - z = -e^t + \frac{1}{6}t^3 - At^2 + (1 - B)t - 2A - C.$$

Preintegrovanim dostaneme

$$y(t) = -e^t + \frac{1}{24}t^4 - \frac{A}{3}t^3 + \frac{1 - B}{2}t^2 - (2A + C)t + D,\ \ D \in \mathbb{R},\ \ t \in \mathbb{R}.$$

(δ)

$$\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} -e^t + \frac{1}{24}t^4 - \frac{A}{3}t^3 + \frac{1-B}{2}t^2 - (2A + C)t + D \\ e^t - \frac{1}{6}t^3 + At^2 + Bt + C \end{pmatrix},\ A, B, C, D, t \in \mathbb{R}.$$

1.5

(α)

Resime soustavu

$$\begin{aligned} u' &= v + w, \\ v' &= u + w, \\ w' &= u + v. \end{aligned}$$

Odpovidajici λ-matice tedy je $\left(\begin{array}{ccc|c} \lambda & -1 & -1 & 0 \\ -1 & \lambda & -1 & 0 \\ -1 & -1 & \lambda & 0 \end{array}\right)$. Navic chceme, aby reseni splnovalo pocatecni podminku $\begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$.

(β)

$$\begin{array}{r} III: \\ II - III: \\ I + \lambda III: \end{array} \left(\begin{array}{ccc|c} -1 & -1 & \lambda & 0 \\ 0 & \lambda+1 & -\lambda-1 & 0 \\ 0 & -\lambda-1 & \lambda^2-1 & 0 \end{array}\right),$$

$$\begin{array}{r} I: \\ II: \\ II + III: \end{array} \left(\begin{array}{ccc|c} -1 & -1 & \lambda & 0 \\ 0 & \lambda+1 & -\lambda-1 & 0 \\ 0 & 0 & \lambda^2-\lambda-2 & 0 \end{array}\right).$$

(γ)

(1) Vypocet $w(t)$: Z 3. radku vidime, ze w splnuje nasledujici rovnici

$$w'' - w' - 2w = 0.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice vyssiho radu.

(i) $\chi(t) = t^2 - t - 2$,

(ii) $\{-1, 2\}$,

(iii) $\left\{e^{-t}, e^{2t}\right\}$,

(iv) $w(t) = Ae^{-t} + Be^{2t}$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$.

(2) Vypocet $v(t)$: Z 2. radku vidime, ze v splnuje nasledujici rovnici

$$v' + v = w' + w = 3Be^{2t}.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice 1. radu $v' = p(t)v + q(t)$.

(i) $p(t) = -1$, $q(t) = 3Be^{2t}$.

(ii)

$$P(t) = \int p(t)dt = \int -1dt = -t,$$

$$K(t) = \int q(t)e^{-P(t)}dt = \int 3Be^{3t}dt = Be^{3t}.$$

(iii) $v(t) = e^{P(t)}(K(t) + C) = Ce^{-t} + Be^{2t}$, $C \in \mathbb{R}$, $t \in \mathbb{R}$.

(3) Vypocet $u(t)$: Z 1. radku vidime, ze

$$u(t) = -v + w' = -(C + A)e^{-t} + Be^{2t}.$$

(δ)

$\begin{pmatrix} u(t) \\ v(t) \\ w(t) \end{pmatrix} = e^{-t} \begin{pmatrix} -C-A \\ C \\ A \end{pmatrix} + e^{2t} \begin{pmatrix} B \\ B \\ B \end{pmatrix}$, $A, B, C \in \mathbb{R}$, $t \in \mathbb{R}$. Pocatecni podminka: $\begin{pmatrix} u(0) \\ v(0) \\ w(0) \end{pmatrix} = \begin{pmatrix} -A+B-C \\ B+C \\ A+B \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$. Tedy $A = B = 0$ a $C = 1$.

1.6

(α)

Resime soustavu $(x', y', z')^T = \mathbb{A}(x, y, z)^T + \boldsymbol{b}(t)$, kde

$$\mathbb{A} = \begin{pmatrix} 0 & 1 & 0 \\ -4 & 4 & 0 \\ -2 & 1 & 2 \end{pmatrix}, \boldsymbol{b}(t) = \begin{pmatrix} 1 \\ t \\ t^2 \end{pmatrix}.$$

$$(\lambda\mathbb{I} - \mathbb{A}|\boldsymbol{b}(t)) = \left(\begin{array}{ccc|c} \lambda & -1 & 0 & 1 \\ 4 & \lambda-4 & 0 & t \\ 2 & -1 & \lambda-2 & t^2 \end{array}\right).$$

(β)

$$\begin{array}{r} III: \\ II-2III: \\ 2I-\lambda III: \end{array} \left(\begin{array}{ccc|c} 2 & -1 & \lambda-2 & t^2 \\ 0 & \lambda-2 & 4-2\lambda & t-2t^2 \\ 0 & \lambda-2 & -\lambda^2+2\lambda & 2-2t \end{array}\right),$$

$$\begin{array}{r} I: \\ II: \\ II-III: \end{array} \left(\begin{array}{ccc|c} 2 & -1 & \lambda-2 & t^2 \\ 0 & \lambda-2 & 4-2\lambda & t-2t^2 \\ 0 & 0 & \lambda^2-4\lambda+4 & -2t^2+3t-2 \end{array}\right).$$

(γ)

(1) Vypocet $z(t)$: Z 3. radku vidime, ze z splnuje nasledujici rovnici

$$z'' - 4z' + 4z = -2t^2 + 3t - 2.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice vyssiho radu.

(i) $\chi(t) = t^2 - 4t + 4$,

(ii) $\{2, 2\}$,

(iii) $\{e^{2t}, te^{2t}\}$,

(iv) $z_h(t) = e^{2t}(A + Bt)$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$.

(v) $m = 0, \mu = 0, \nu = 0, k = 2$.

(vi) $R(t) = at^2 + bt + c$, $z_p(t) = at^2 + bt + c$, kde $a, b, c \in \mathbb{R}$, $t \in \mathbb{R}$.

(vii) $a = -\frac{1}{2}$, $b = -\frac{1}{4}$, $c = -\frac{1}{2}$.

(xiii) $z(t) = e^{2t}(A + Bt) - \frac{1}{4}(2t^2 + t + 2)$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$.

(2) Vypocet $y(t)$: Z 2. radku vidime, ze y splnuje nasledujici rovnici

$$y' - 2y = t - 2t^2 + 2z' - 4z = \frac{3}{2} + 2Be^{2t}.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice 1. radu $y' = p(t)y + q(t)$.

(i) $p(t) = 2$, $q(t) = \frac{3}{2} + 2Be^{2t}$.

(ii)

$$P(t) = \int p(t)dt = \int 2dt = 2t,$$

$$K(t) = \int q(t)e^{-P(t)}dt = \int \frac{3}{2}e^{-2t} + 2Bdt = -\frac{3}{4}e^{-2t} + 2Bt.$$

(iii) $y(t) = e^{P(t)}(K(t) + C) = e^{2t}(C + 2Bt) - \frac{3}{4}$, $C \in \mathbb{R}$, $t \in \mathbb{R}$.

(3) Vypocet $x(t)$: Z 1. radku vidime, ze

$$x(t) = \frac{1}{2}\left(t^2 + y - z' + 2z\right) = \frac{1}{4}(t-3) + \frac{1}{2}e^{2t}(C - B + 2Bt).$$

(δ)

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = e^{2t}\begin{pmatrix} \frac{C-B+2Bt}{2} \\ C+2Bt \\ A+Bt \end{pmatrix} + \begin{pmatrix} \frac{t-3}{4} \\ -\frac{3}{4} \\ -\frac{2t^2+t+2}{4} \end{pmatrix}, \ A, B, C \in \mathbb{R}, \ t \in \mathbb{R}.$$

1.7

(α)

Resime soustavu $(x', y', z')^T = \mathbb{A}(x, y, z)^T + \boldsymbol{b}(t)$, kde

$$\mathbb{A} = \begin{pmatrix} 2 & 6 & -15 \\ 1 & 1 & -5 \\ 1 & 2 & -6 \end{pmatrix}, \boldsymbol{b}(t) = \begin{pmatrix} e^t \\ e^t \\ e^t \end{pmatrix}.$$

$$(\lambda\mathbb{I} - \mathbb{A}|\boldsymbol{b}(t)) = \left(\begin{array}{ccc|c} \lambda-2 & -6 & 15 & e^t \\ -1 & \lambda-1 & 5 & e^t \\ -1 & -2 & \lambda+6 & e^t \end{array}\right).$$

(β)

$$\begin{array}{r} III: \\ II - III: \\ I + (\lambda-2)III: \end{array} \left(\begin{array}{ccc|c} -1 & -2 & \lambda+6 & e^t \\ 0 & \lambda+1 & -1-\lambda & 0 \\ 0 & -2\lambda-2 & \lambda^2+4\lambda+3 & 0 \end{array}\right),$$

$$\begin{array}{r} I: \\ II: \\ III + 2II: \end{array} \left(\begin{array}{ccc|c} -1 & -2 & \lambda+6 & e^t \\ 0 & \lambda+1 & -1-\lambda & 0 \\ 0 & 0 & \lambda^2+2\lambda+1 & 0 \end{array}\right).$$

(γ)

(1) Vypocet $z(t)$: Z 3. radku vidime, ze z splnuje nasledujici rovnici

$$z'' + 2z' + z = 0.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice vyssiho radu.

(i) $\chi(t) = t^2 + 2t + 1$,

(ii) $\{-1, -1\}$,

(iii) $\{e^{-t}, te^{-t}\}$,

(iv) $z(t) = e^{-t}(A + Bt)$, $A, B \in \mathbb{R}$, $t \in \mathbb{R}$.

(2) Vypocet $y(t)$: Z 2. radku vidime, ze y splnuje nasledujici rovnici

$$y' + y = z' + z = Be^{-t}.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice 1. radu $y' = p(t)y + q(t)$.

(i) $p(t) = -1$, $q(t) = Be^{-t}$.

(ii)

$$P(t) = \int p(t)dt = \int -1dt = -t,$$

$$K(t) = \int q(t)e^{-P(t)}dt = \int Bdt = Bt.$$

(iii) $y(t) = e^{P(t)}(K(t) + C) = e^{-t}(C + Bt)$, $C \in \mathbb{R}$, $t \in \mathbb{R}$.

(3) Vypocet $x(t)$: Z 1. radku vidime, ze

$$x(t) = -2y + z' + 6z - e^t = e^{-t}(5A + B - 2C + 3Bt) - e^t.$$

(δ)

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = e^{-t} \begin{pmatrix} 5A + B - 2C + 3Bt \\ C + Bt \\ A + Bt \end{pmatrix} + e^t \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}, \ A, B, C \in \mathbb{R}, \ t \in \mathbb{R}.$$

1.8

(α)

Resime soustavu $(x', y', z', w')^T = \mathbb{A}(x, y, z, w)^T$, kde

$$\mathbb{A} = \begin{pmatrix} 1 & -3 & 0 & 3 \\ -2 & -6 & 0 & 13 \\ 0 & -3 & 1 & 3 \\ -1 & -4 & 0 & 8 \end{pmatrix}.$$

Odpovidajici λ-matice tedy je $\left(\begin{array}{cccc|c} \lambda - 1 & 3 & 0 & -3 & 0 \\ 2 & \lambda + 6 & 0 & -13 & 0 \\ 0 & 3 & \lambda - 1 & -3 & 0 \\ 1 & 4 & 0 & \lambda - 8 & 0 \end{array}\right)$.

(β)

$$\begin{array}{r} I - (\lambda - 1)IV: \\ II - 2IV: \\ III: \\ IV: \end{array} \left(\begin{array}{cccc|c} 0 & 7 - 4\lambda & 0 & -\lambda^2 + 9\lambda - 11 & 0 \\ 0 & \lambda - 2 & 0 & -2\lambda + 3 & 0 \\ 0 & 3 & \lambda - 1 & -3 & 0 \\ 1 & 4 & 0 & \lambda - 8 & 0 \end{array}\right),$$

$$\begin{array}{r} I + 4II: \\ II: \\ III: \\ IV: \end{array} \left(\begin{array}{cccc|c} 0 & -1 & 0 & -\lambda^2 + \lambda + 1 & 0 \\ 0 & \lambda - 2 & 0 & -2\lambda + 3 & 0 \\ 0 & 3 & \lambda - 1 & -3 & 0 \\ 1 & 4 & 0 & \lambda - 8 & 0 \end{array}\right),$$

$$\begin{array}{r} I: \\ II + (\lambda - 2)I: \\ III: \\ IV: \end{array} \left(\begin{array}{cccc|c} 0 & -1 & 0 & -\lambda^2 + \lambda + 1 & 0 \\ 0 & 0 & 0 & -(\lambda - 1)^3 & 0 \\ 0 & 3 & \lambda - 1 & -3 & 0 \\ 1 & 4 & 0 & \lambda - 8 & 0 \end{array}\right).$$

(γ)

(1) Vypocet $w(t)$: Z 2. radku vidime, ze w splnuje nasledujici rovnici

$$z''' + 3w'' - 3w' + w = 0.$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice vyssiho radu.

(i) $\chi(t) = (t-1)^3$,

(ii) $\{1, 1, 1\}$,

(iii) $\{e^t, te^t, t^2e^t\}$,

(iv) $w(t) = e^t(A + Bt + Ct^2)$, $A, B, C \in \mathbb{R}$, $t \in \mathbb{R}$.

(2) Vypocet $y(t)$: Z 1. radku vidime, ze

$$y(t) = -w'' + w' + w = e^t(A - B - 2C + (B - 2C)t + Ct^2).$$

(3) Vypocet $z(t)$: Z 3. radku vidime, ze z splnuje nasledujici rovnici

$$z' - z = -3y + 3w = 3e^t(B + 2C + 2Ct).$$

Tuto rovnici vypocteme podle postupu pro reseni linearni diferencialni rovnice 1. radu $z' = p(t)z + q(t)$.

(i) $p(t) = 1$, $q(t) = 3e^t(B + 2C + 2Ct)$.

(ii)

$$P(t) = \int p(t)dt = \int 1dt = t,$$

$$K(t) = \int q(t)e^{-P(t)}dt = \int 3(B + 2C + 2Ct)dt = (3B + 6C)t + 3Ct^2.$$

(iii) $z(t) = e^{P(t)}(K(t) + D) = e^t(D + (3B + 6C)t + 3Ct^2)$, $D \in \mathbb{R}$, $t \in \mathbb{R}$.

(4) Vypocet $x(t)$: Z 4. radku vidime, ze

$$x(t) = -4y - w' + 8w = e^t(3A + 3B + 8C + t(3B + 6C) + 3Ct^2).$$

(δ)

$$\begin{pmatrix} x(t) \\ y(t) \\ z(t) \\ w(t) \end{pmatrix} = e^t \begin{pmatrix} 3A + 3B + 8C + t(3B + 6C) + 3Ct^2 \\ A - B - 2C + (B - 2C)t + Ct^2 \\ D + (3B + 6C)t + 3Ct^2 \\ A + Bt + Ct^2 \end{pmatrix}, \ A, B, C, D \in \mathbb{R},\ t \in \mathbb{R}.$$